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SOLUTION OF ONE PROBLEM ON OPTIMUM GAS WELL OPERATION CONTROL

Abstract.*The article is devoted to the numerical solution of the problem of optimum managing gas well operation modes. The solution to the problem is based on the approximation of a partial differential equation by a system of ordinary differential equations. Particular attention is paid to the numerical solution of the optimal control problem associated with these systems based on the Pontryagin's maximum principle. To solve the problem, a linearization method and implicit finite difference schemes for solving a nonlinear equation are proposed. The calculation of technological modes of wells operation by adjusting the bottomhole pressure within certain limits is based on the results of theoretical and experimental studies.*

Keywords*: Technological mode, Bottomhole pressure, the method of straight lines, Gradient projection method.*

JEL Classification: C610, C630, C680.

1. Introduction

In modern conditions characterized by rising energy prices, the efficiency and cost-effectiveness of the operation of gas fields and underground gas storages are important factors in reducing costs and increasing the reliability of gas supplies to consumers. An important scientific and technical problem of field development is to ensure high levels and rates of hydrocarbon production with the most complete extraction from the bowels, as well as the high technical and economic

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performance of gas-producing enterprises. Therefore, improving the technology for developing gas and gas condensate fields is an urgent and important task for the oil and gas industry.

The goal of this work is to improve the quality of numerical solutions to optimal control problems for nonlinear systems with distributed parameters through the Pontryagin's maximum principle. This goal is achieved without the need to consider the conjugate boundary value problem for partial differential equations.

All of the above determined the structure of the work, which consists of six main parts. The introduction substantiates the relevance of the topic and formulates the objectives of the study. After the introduction, the methodology describes the applied research methods, substantiates the scientific novelty and practical value of solving the problem under consideration. Further, the work contains a review that surveys the literature in the chosen area of study. The main part of the study includes the theoretical and practical part of constructing the mathematical apparatus of the optimal control problem and determining the technological mode of gas well operation. The study is completed by discussion and conclusion, constituting findings on the research.

2. Research methodology

Many articles are devoted to the optimal control of wells' technological regimes. The scientific novelty of this work lies in the development and justification of a method on solving the optimal control problem for systems with distributed parameters. In the problem considered, the technological mode of operation of gas wells is determined by regulating the bottomhole pressure within certain limits. The proposed approximate solution of the posed boundary value problem is found using the straight-line method.

The purpose of the paper is to prove convergence on functional for which the approximate optimal control is found minimizing. The gradient projection method is used with a special choice of step, and there is no tendency to "hopper agitation". In a short time, a convergent minimizing sequence is obtained in the control space.

When performing the work, methods of the theory of optimal control and linear programming, various linearization methods, and implicit finite difference schemes for solving the nonlinear equation are used. The experimental material is based on calculations performed using actual data on gas storage facilities, as well as on the experience of implementing technical solutions developed and justified in this work.

The validity and reliability of the results of the manuscript are ensured by the correctness and completeness of the models used, the convergence of computational algorithms, the results of testing algorithms and programs, and experimental studies.

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3. Literature review

As part of gas production technology, gas wells are the most numerous objects. Complicating conditions for gas production, increasing requirements for the quality of gas field management, improving field development indicators have led to the urgent need for more active implementation of automated gas well management as part of information-measuring and control systems.

However, research studies have not yet sufficiently reflected the decision-making methods for managing gas wells. The complexity of this task lies not only in a large number of wells at one control object but also in the interdependence of the wells' operation. This interdependence remains notoriously complex: the interaction between the wells occurs both through the gas collection and transportation system, and through the gas-bearing stratum. Also, there are factors of uncertainty in the parameters of these objects. Note that after the emergence of the famous Pontryagin's maximum principle [14] optimization methods have found wide applications in modelling and solving various problems of developing oil and gas fields.

To obtain the characteristics of the regimes for maintaining the maximum allowable pressure gradient on the walls of the wells, Zakirov [23] uses two equations - the nonlinear law of resistance for the filtration rate and the equation of gas inflow to the bottom of the well with a nonlinear law of resistance.

In the paper [8] the problem of finding a rational option for the development of a gas field is considered and an algorithm for its solution using the gradient method is proposed. An example of solving the problem for a typical gas field in Western Siberia is given.The article [3] considers the task of determining the technological modes of well operation. According to the authors, these modes provide optimal technical and economic indicators of development and the most complete extraction of oil or gas from the bowels. The problem with the condition of two-phase filtration is reduced to the task of optimal control of filtration flows in the reservoir.In survey paper [17] the problem of mathematical modelling of the process of extracting oil from heterogeneous formations and methods of solution are considered. The task of optimal regulation of the oil recovery process is posed and an assessment of the main factors affecting this process is given. In the manuscript [1] the problem of optimal placement of oil reservoir wells and flow rate management is investigated. Mathematically, this problem is a parametric task of optimal control of a distributed system concentrated by sources, described by differential equations in partial derivatives.

In his next work, Zakirov [24] considers the problem of maximizing oil production from a multilayer field with a single grid of production wells and separate grids of injection wells. To solve the problem, methods of the theory of optimal control are used. In the paper [16] the model of the functioning of a gas field with interrelated wells is investigated. The optimal control problem is posed and solved on an infinite interval. Tugov in [20] describes the search for the

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optimal control action on the oil mixture in a primary preparation separation unit using the acoustic treatment. The application of the maximum principle, which allows determining the values of the control actions on the oil mixture, is considered, experiments to study the effect of ultrasonic exposure on the separation process are conducted under laboratory conditions.

It should be noted that in the two last decades, completely new approaches based on the use of genetic algorithms and neural networks have appeared to solve optimization problems associated with the design and development of gas and oil fields. For example, a study [22] aims to represent the body of knowledge on evolutionary computing used in the field of exploration and production in the oil and gas industry. It also describes the structure of evolutionary engineering systems in determining reservoir characteristics. In the manuscript [25] many various methods are compared with work of genetic algorithms in the field we are studying. The authors investigate the ant colony optimization (ACO) method, which has shown excellent properties in optimizing water distribution networks. The study [12] presents a new methodology for predicting fluid flow behavior using the artificial neural network. The developed methodology allows predicting a wide variety of cost items, ranging from the pressure at the inlet to the throttle up to the size of the throttle. The accuracy of the developed model is proved by empirical correlations.

Soemardan in [18] develops economics-mathematical model for optimizing gas production on the example of research of the Matindok field. The authors analyse marginal costs in determining the optimal level of gas production. The results obtained show that the optimal resource extraction rate is in direct proportion to its price and transportation time.Namdar in [13] claims that the increased speed and accuracy in solving gas distribution optimization problems are determined by the nature of the allocation of either form of reef structures and the structural-tectonic factor, including the presence of high-amplitude shafts and flexures. The optimization solution consists of two successive stages: (1) fitting the gas lift productivity curve (gas lift modelling) and (2) optimizing the gas distribution between the wells. The results obtained allow substantiating the optimal technology for conducting research and interpreting the results to diagnose the proportion of cracks in the tributary with the goal of uniform reservoir development. Janiga in[6] describes the non-uniformity of pressure reduction in interlayers with different filtration-capacitive properties during the development of a gas condensate field. The result of an uneven pressure reduction in the reservoir is the occurrence of interstratal flows of the gas-condensate mixture even in the presence of a slight hydrodynamic contact between the layers. The proposed approach to the restoration of reservoir properties in the interwell space using reference points has been tested on synthetic models. It is shown that the application of the proposed approach allows saving geological information in the process of refining the model.

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The article [4] presents the modern achievements of gas lift technology and promising directions for the development of this method for well operation. The authors consider the aspects of establishing the necessary technological mode of operation of the wells, taking into account the current state of the reservoir-well system, as well as the planning of geological and technical measures aimed at intensifying the selection of reservoir fluids. To analyse the information, physical and mathematical models are used in the online mode; this makes it possible to identify the dynamic features of changes in the analysed parameters and the presence of periodic self-organizing processes.

Purification of natural and other gases from hydrogen sulphide can be carried out by different methods. The choice of the process of natural gas purification from sulphur compounds in each case depends on many factors, the main of which are: the composition and parameters of the feed gas, the required degree of purification and the use of commercial gas, the availability and parameters of energy resources, production waste, etc. An analysis of world practice accumulated in the field of natural gas purification shows that absorption processes for processing large gas flows using chemical and physical absorbents and their combinations are the main ones. Shang in[15] tests a stationary simulation modelling the process of natural gas purification from high sulphur content using ProMax. Using the back propagation neural network based on the analysis of the integrated distribution of energy consumption, seven main operating parameters of the cleaning process are determined. The article [2] uses the PSA - particle swarm algorithm to determine the intervals for filling wells between sampling points in a synthetic reservoir with constant fluid measurement. Real-time image registration provides the necessary information about the structure of the collector and, ultimately, helps to keep the trajectory in the most productive zone. The objective function in this study is the net present value of the asset (reservoir). The effective Monte-Carlo method presented in the article [7] is an optimization plan of the gas condensate field development, taking into account the distribution of fluid backstops in terms of reliability, and allows minimizing geological risks and uncertainties during well construction. The work studies the solubility kinetics of various types of clays in acidic compositions and their components depending on the concentration of reagents, temperature, and duration of the experiment.

Despite a lot of research in this direction, the known methods are not adapted to develop optimal solutions for managing gas wells in real-time. Thus, the control and management of gas wells, taking into account the interaction of wells with other elements of the gas production technological complex in the face of parameters' uncertainty, remains an urgent task.

4. Problem statement

The choosing technological mode of gas well operation is one of the most important decisions made during the mining and management of field

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development. Generally, the technological regime of well operation is understood as the regulation of the well flowing $q(t)$ or pressure $p_c(t)$ in the bottomhole zone. Conditions ensuring compliance with the rules relating protection of mineral resources and trouble-free operation of wells are supported with their help. One of the simplest technological modes of gas well operation is the maximum allowable depression mode. With growing depression, the flowing of the production well increases. This mode is mathematically written as$p_{п}(t) - p_c(t) = \delta$, where $p_{п}(t)$ is the reservoir pressure in the zone of some well at time t; $p_c(t)$ is the bottomhole pressure in the same well at time t; δ is the allowable depression on the reservoir.

In this article, the technological mode of gas well operation is determined by adjusting the bottomhole pressure $p_c(t)$ within certain limits. This task can be attributed to the class of optimal control problems for systems with distributed parameters. Relatively dimensionless quantities, it can be formulated as follows: operate the pressure $p_c(t)$, satisfying the inequality

$$0 < p_1 \leq p_c(t) \leq p_2 \leq 1, \tag{1}$$

in the time interval $0 \leq t \leq T$, where p_1 and p_2 are constants defined on the basis of technical and economic calculations so that the amount of gas produced from wells minimally deviates from its previously planned value $q^*(t)$. A quadratic functional is taken as a measure of such a deviation

$$F = \frac{1}{2}\int_0^T \left[\frac{\partial p^2(0,t)}{\partial x} - q^*(t)\right]^2 dt \tag{2}$$

Here $p(x,t)$ describes the distribution of gas pressure in the “reservoir” $0 \leq x \leq 1$, which, with the linear law of filtration, is a solution of the non-linear Leibenzon equation [9]:

$$\frac{\partial p}{\partial t} = \frac{1}{2} \cdot \frac{\partial^2 p^2}{\partial x^2}, \tag{3}$$

under the following boundary conditions

$$p(x,0) = \text{const} = 1, \ \ 0 \leq x \leq 1, \tag{4}$$

$$p(0,t) = p_c(t), \ \ \frac{\partial p(1,t)}{\partial x} = 0, \ \ 0 < t \leq T, \tag{5}$$

where $p_c(t)$ is a piecewise continuous function in the interval $0 \leq t \leq T$, and conditions (4) and the first condition in (5) are consistent: $p_c(0) = 1$.

Condition (4) is the initial one and means that at the initial moment of time the distribution law of the reservoir pressure is known. The second condition in (5) indicates the impermeability of the external boundary of the reservoir. Note that when solving some filtration problems at the outer boundary $x = 1$, the reservoir

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pressure value $p_{\text{п}}(t)$ is set, that is, the second condition in (5) can be replaced by the condition $p(1,t) = p_{\text{п}}(t)$. Consider the differential equation

$$\frac{dy}{dt} = \frac{1}{2}\left[\frac{\partial p^2(0,t)}{\partial x} - q^*(t)\right]^2, \quad y(0) = 0 \tag{6}$$

related to functional (2). Then functional (2) is written as

$$F = y(T). \tag{7}$$

Thus, problem (1) – (5) is narrowed down to the problem of systems' optimal control, the behavior of which is described by a set of differential equations in partial and ordinary derivatives. We note that optimal control problems associated with a more general boundary-value problem similar to (3) – (6) were first considered by Egorov [3]in the mid-60s of the last century, and the early 70s and subsequent years were the subject of research by many Russian and foreign authors. Further, as such practically important problems appear the activation of research in this direction is reflected in [19]. In the paper [5]the control problem of the so-called noisy dynamic systems associated with obtaining an assessment of the state and parameters of the control object is considered. In the manuscript [3] the problem of dumping the oscillations of a system described by a combination of a wave equation and an ordinary differential equation of the second order is considered under the assumption that the control function and the object with lumped parameters act, respectively, on the left and right ends of the object with distributed parameters. The functions of the states of the system are connected through the boundary conditions for the wave equation. To solve the problem, the d'Alembert formula was used and, applying the method of straight lines, finite-dimensional approximations of the problem were constructed. Teymurov in [19] investigated the problem of optimal control of processes described by a combination of parabolic type equations and ordinary differential equations with controls of moving sources. The existence and uniqueness theorem of the solution was proved, the necessary optimality conditions were obtained in the form of point and integral maximum principles.

It is easy to see that when solving problems related to regulation in a given range of well production, ensuring depletion of a gas reservoir by a given point in time, the first condition in (5) should be replaced by the condition

$$\frac{\partial p^2(0,t)}{\partial x} = q(t), \ 0 < q_1 \le q(t) \le q_2, \ 0 < t \le T, \tag{8}$$

and instead of (2) minimize the functional

$$F = \frac{1}{2}\int_0^1 [\,p(x,T) - p^*(x)]^2 dx, \tag{9}$$

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where $p^*(x)$ is the gas pressure over the reservoir, specified on the basis of technological considerations, q_1, q_2 are constant values. Condition (8) shows that a well located at the "point" $x = 0$ is operated with a production rate $q(t)$. A numerical solution to this problem was obtained in [11]. Instead of (2) we consider minimization of the functional

$$F = \frac{1}{2}\int_0^T [\, p(0,t) - p_3^*(t) - \delta\,]^2 dx, \qquad (10)$$

taking into account phase restrictions

$$p(0,t) > p_3^*(t), \qquad (11)$$

where $p_3^*(t)$ is the specified pressure in the bottomhole zone of the well, and δ is the allowable depression on the formation and is a given number. Then we have to deal with the control task of choosing the technological mode of operation of gas wells that provides the maximum allowable depression on the formation [24].

5. Numerical solution of problem (1), (3) – (7)

Due to the impossibility of obtaining an analytical solution of the boundary value problem (3) – (6), although in [9] various linearization methods and implicit finite-difference schemes for solving the nonlinear equation (3) are proposed. An approximate solution to the boundary value problem (3) – (6) will be sought by the straight line method, replacing it at the grid nodes of the lines $x_i = ih, i = 1,2,\dots,n,$ $(n+1)h = 1,$ $x_0 = 0,$ $x_{n+1} = 1$ by the system of differential-difference equations:

$$\frac{dz_i}{dt} = \frac{1}{2h^2}\left[z_{i-1}^2 - 2z_i^2 + z_{i+1}^2\right], \ \ i = 1,2,\dots,n, \ \ z_0 = p_c(t), \ \ z_{n+1} = z_n, \qquad (12)$$

$$\frac{dy_n}{dt} = \frac{1}{2h^2}[z_1^2 - p_c^2 - hq^*(t)\,]^2$$

with initial conditions

$$z_i(0) = 1, \ \ i = 1,2,\dots,n, \ \ y_n(0) = 0, \qquad (13)$$

where $z_i(t) = p(x_i,t), \ \ y_n(t) = y(t), \ \ i = 1,2,\dots,n.$

The system of differential-difference equations (12) is valid for all internal nodal points, $x_i = ih, \ i = 1,2,\dots,n,$, where h is the step along the spatial coordinate.The approximating functional has the form:

$$F = y_n(T) \qquad (14)$$

Therefore, using the method of straight lines, problem (1), (3) – (7) reduces to the optimal control problem for concentrated systems with a free right end [14].

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Using a priori estimates known for systems of linear ordinary differential equations, it is easy to prove that the solution of the differential-difference system (12)– (13) converges as $h \to 0$ with the speed $O(h)$ to the solution of the boundary value problem (3) - (6), and functional convergence takes place.

We write the system of conjugate equations:

$$\frac{d\psi_1}{dt} = -\frac{\partial H}{\partial z_1} = -\frac{z_1}{h^2}[-2\psi_1 + \psi_2 - 2(z_1^2 - p_c^2 - hq^*(t))\varphi_n],$$
$$\frac{d\psi_i}{dt} = -\frac{\partial H}{\partial z_i} = -\frac{z_i}{h^2}[\psi_{i-1} - 2\psi_i + \psi_{i+1}],\ i = 2, \dots, n-1, \tag{15}$$
$$\frac{d\psi_n}{dt} = -\frac{\partial H}{\partial z_n} = -\frac{z_n}{h^2}[\psi_{n-1} - \psi_n], \frac{d\varphi_n}{dt} = -\frac{\partial H}{\partial y_n} = 0$$

with conditions at the right end

$$\psi_i(T) = 0,\ i = 1,2, \dots, n, \quad \varphi_n(T) = 1, \tag{16}$$

where

$$H = \frac{1}{2h^2}\left\{\sum_{i=1}^{n} \psi_i\left[z_{i-1}^2 - 2z_i^2 + z_{i+1}^2\right] + \varphi_n[z_1^2 - p_c^2 - hq^*(t)\ ]^2\right\} \tag{17}$$

is Hamilton - Pontryagin function of problem (1), (12) – (14).

Note that the system of equations (15) – (16) with ordinary derivatives can be obtained directly and approximating the boundary value problem conjugate for (3) – (6) by the method of straight lines

$$\frac{\partial \psi}{\partial t} = -p(x,t) \cdot \frac{\partial^2 \psi}{\partial x^2}, \tag{18}$$
$$\psi(x,T) = 0,\ 0 \le x \le 1, \tag{19}$$
$$\psi(0,t) = -2\varphi(t)\left[\frac{\partial p^2(0,t)}{\partial x} - q^*(t)\right], \frac{\partial \psi(1,t)}{\partial x} = 0,\ 0 \le t < T, \tag{20}$$
$$\frac{d\varphi}{dt} = 0,\ \varphi(T) = 1,\ 0 \le t < T, \tag{21}$$

compiled on the basis of the stated in [21] general approach used in deriving the formula for the gradient of functional (7). However, many authors prefer to consider the conjugate problem precisely for partial differential equations, since in this case the initial distributed system can be solved not only by the method of straight lines but also by any numerical methods, in particular, by an implicit difference scheme in combination with “walk-through”[21].

Considering that $\varphi_n(t) \equiv \text{const} = 1$, from (17) we have:

$$\frac{\partial H}{\partial p_c} = \frac{1}{h^2}\{\psi_1 - 2[\ z_1^2 - p_c^2 - hq^*(t)]\}p_c \tag{22}$$

To numerically solve problem (1), (12) - (14), we choose some initial control $p_c^0(t)$ that satisfies (1), taking into account the conditions $p_c(0) = 1$. The

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Runge – Kutta method solves the Cauchy problem for the system of equations (12) – (13) and finds the values of the functions $z_i(t), i = 1,2, \ldots, n,\ y_n(t)$ in the time interval $0 \le t \le T$ and their values are remembered. Then, in the "opposite direction" of time, the Cauchy problem for the conjugate system (15) - (16) is solved, the coefficients of which are calculated along with the trajectories $z_i(t), y_n(t)$ and at each step of integration the values $\partial H/\partial p_c$are found. Further, new, improved controls at successive approximations are calculated by the formulas

$$p_c^{k+1}(t) = \begin{cases} p_c^k(t) - \delta p_c^k(t), & \text{if}\, p_1 < \ p_c^k(t) - \delta p_c^k(t) < p_{2,} \\ p_1, & \text{if}\ p_c^k(t) - \delta p_c^k(t) \le p_{1,} \\ p_2, & \text{if}\ p_c^k(t) - \delta p_c^k(t) \ge p_2 \end{cases} \tag{23}$$

taking into account the conditions $p_c^k(0) = 1$, where $\delta p_c^k(t)$is the control improvement change, which for a problem with a fixed left end, a free right end and in the absence of restrictions is usually constructed according to the scheme $\delta p_c^k(t) = -\lambda_k F'(p_c^k(t))$.In the paper [10] control is calculated in a special form according to the rule

$$\delta p_c^k(t) = \lambda \cdot \frac{\partial H^k(t)/\partial p_c}{|\partial H_0^k(t)/\partial p_c|}\ , \quad k = 0,1,2, \ldots \tag{24}$$

Here k is the iteration number, $H_0/\partial p_c$ is the maximum value for $\partial H/\partial p_c$, taken in absolute value for $0 \le t \le T$, and $\lambda > 0$ is the step size. Depending on the selection method, as a rule, various forms of first-order gradient methods are obtained. The iterative process (23) – (24) continues until one of the described in [21] criteria for ending the count are fulfilled; sometimes the number of iterations is predefined. To implement the above scheme, the program is compiled in QBasic.

The initial values of the parameters, one-dimensional arrays for storing function values $p_c^0(t)$, $p_c^1(t)$, $q^*(t)$,$\partial H/\partial p_c$, $\delta p_c^k(t)$ and two two-dimensional arrays for solutions of the approximating and conjugate systems are entered in the initial lines of the program. In the next lines, the system is approximated, the functional values are calculated, and a system of conjugate equations is solved in the opposite direction of time. For clarity, the following few lines of the program indicate the commands designed to determine the maximum element of the array $\partial H/\partial p_c$, calculate the new control $p_c^1(t)$ and print the value of the functional, as well as the solution to the approximating system (12) – (13), corresponding to this control.

```
..................................................
370 max = dhdp (0)
380 for j =1 to n
390 if max <dhdp (j) then max = dhdp (j)
400 next j
495 rem array calculation dp0 (n)
```


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```
510 for j = 0 to n
520 dp0 (j) = p0 (j) – lambda*dhdp (j)/abs (max)
530 next j
540 rem calculation of new control
545 p1 (0) = 1
550 for j= 1 to m
560 if dp0 (j) > p2 then p1 (j) = p2 else if dp0 (j) < p1 then p1 (j) = p1
    else p1 (j) = dp0 (j)
570 next j
580 rem output results
584 print "k="; k, "F="; F, "max="; max
586 for j= 0 to m step 2
587 print "p1 ("; j ;") ="; p1 (j)
588 next j
589 for i = 1 to n
590 for j = 0 to m step 5
600 print "z(";i;", ; j;")="; z(i ,j)
610 nexi j
620 next i
630 rem  determination of the termination of the iteration process
..............................................
```

Note that in the process of calculations, as the need arises, some lines can be added to the program, allowing to trace the correctness of the intermediate calculations.

The calculations were performed for the following parameter values:$T = 0.2,\ p_1 = 0.2,\ p_2 = 1$.The function $p_c^0(t) = 1 - 10t^2$ was taken for zero iteration. The segment $0 \leq x \leq 1$ is divided into five parts with a step $h = 0.2$. The systems of equations (12) – (13) and (15) – (16) are integrated with a constant step $t = 0.01$, and the results are output with a step $t = 0.05$. Note that the integration of these systems by the Runge - Kutta method with automatic step selection is associated with some difficulties. Since the values of the functions $z_i(t)$ and $\psi_i(t),\ i = 1,2,\dots,n$ will be calculated at different points, and when integrating system (12) – (13) simultaneously with system (15) – (16) from$t = T$ to $t = 0$ with a constant step, the counting process is often unstable, then the amount of computation increases. To check the optimality found by the control formulas (23) – (24), in the calculations as $q^*(t)$ we took

$$\frac{\partial p^2(0,t)}{\partial x} \approx \frac{(z_1(t))^2 - (p_c^*(t))^2}{h} \tag{25}$$

for a given control $p_c^*(t) = 1 - 4t$. The optimality of $p_c^*(t)$ is obvious since the minimum value of the functional is zero. Note that the found control sequences $p_c^k(t),\ k = 0,1,\dots$ with the increasing number of iterations, as can be seen from the

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above graphs, in the time interval $0 \leq t \leq T$ approach the given control $p_c^*(t)$, and very precisely coincide with the control $p_c^*(t)$. The value of the functional at the 59th iteration turned out to be $\cong$ 6.95854·10^{-7}, and the maximum value of $\partial H/\partial p_c$ turned out to be $\cong$0.0531. With further iterations, the qualitative picture of the results remained practically unchanged.

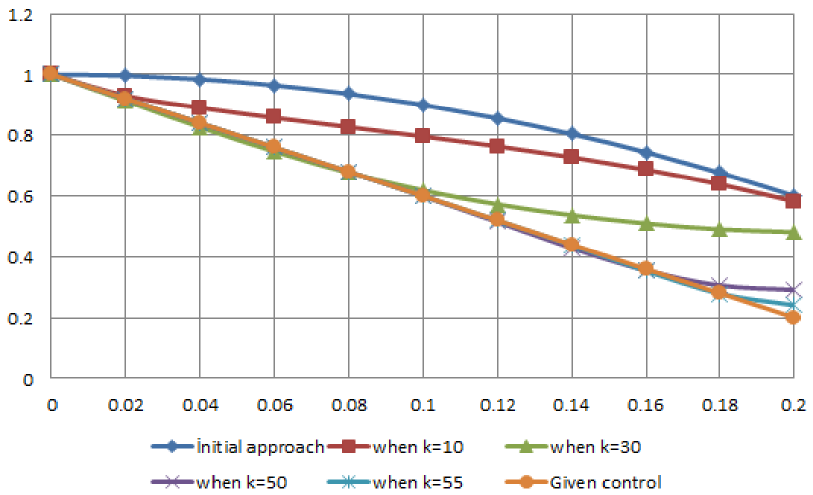

Figure 1. Approximately optimal controls during iterations

Figure 1 shows the obtained minimizing control sequences for some intermediate iterations, and Table 1 shows the convergence in the functional of the iterative process (23) – (24).

Table 1. Convergence in the functional of the iterative process (23) – (24)

k	$y_n(T)$	$\partial H_0/\partial p_c$	k	$y_n(T)$	$\partial H_0/\partial p_c$
0	7.5346·10^{-2}	17.5683	30	8.3990·10^{-3}	3.6220
1	7.1378·10^{-2}	16.7488	40	2.0955·10^{-3}	1.7601
2	6.7571·10^{-2}	16.0118	50	2.1407·10^{-4}	0.6607
3	6.3931·10^{-2}	15.2946	55	2.5792·10^{-5}	0.2793
4	6.0450·10^{-2}	14.5811	56	1.4369·10^{-5}	0.2163
5	5.7115·10^{-2}	13.8980	57	6.0493·10^{-6}	0.1577
10	4.2477·10^{-2}	10.9297	58	2.2042·10^{-6}	0.1034
20	2.1332·10^{-2}	6.5220	59	6.9585·10^{-7}	0.0531

Table 2 shows the calculation results corresponding to the bottomhole pressure $p_c(t)$ found according to the scheme (23)– (24).

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Table 2. The calculation results corresponding to the bottomhole pressure $p_c(t)$ found according to the scheme (23) – (24)

t	$z_0(t) = p_c(t)$	$z_1(t)$	$z_2(t)$	$z_3(t)$	$z_4(t)$	$z_5(t)$
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.02	0.9188	0.9901	1.0000	1.0000	1.0000	1.0000
0.04	0.8405	0.9590	0.9928	0.9993	1.0000	1.0000
0.06	0.7600	0.9209	0.9781	0.9954	0.9993	0.9993
0.08	0.6801	0.8799	0.9586	0.9879	0.9968	0.9968
0.10	0.6001	0.8380	0.9362	0.9773	0.9917	0.9917
0.12	0.5198	0.7961	0.9120	0.9641	0.9840	0.9840
0.14	0.4403	0.7550	0.8868	0.9489	0.9737	0.9737
0.16	0.3598	0.7154	0.8611	0.9320	0.9613	0.9613
0.18	0.2803	0.6778	0.8355	0.9138	0.9470	0.9470
0.20	0.2008	0.6425	0.8102	0.8948	0.9312	0.9312

According to this table, graphs of changes in gas pressure in the reservoir are constructed at different instants of time. Figure 2 shows that the pressure value varies greatly in the area of the bottomhole formation zone, and outside this zone, the pressure graph is represented by an almost straight line. This is due to a violation of the linear law of filtration due to high gas filtration rates in the bottomhole formation zone.

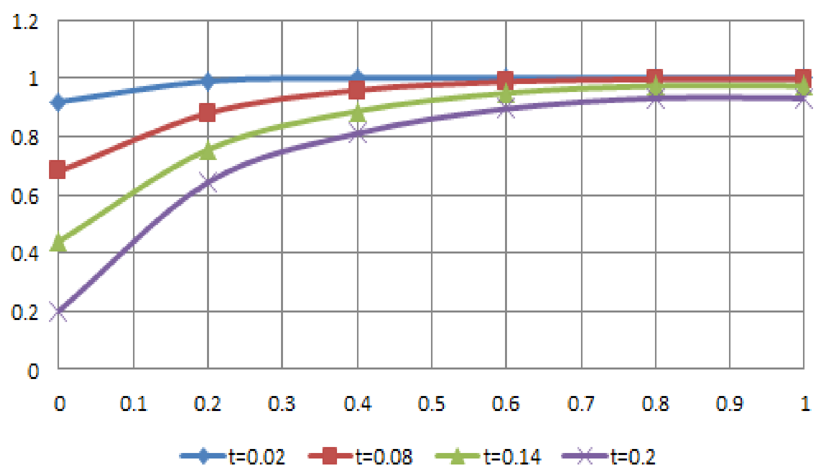

Figure 2. The gas pressure distribution profiles at different instants of time corresponding to the pressure $p_c(t)$found according to scheme (23) – (24)

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Thus, by controlling the bottomhole pressure $p_c(t)$within certain limits, it is possible to maintain conditions in the region of the bottomhole zone that determine the technological mode of gas well operation.

6. Discussion and conclusion

Despite the successes, currently, there are still not sufficiently convincing formulations of the problems on regulating the oil and gas fields' development, although due to their terms these tasks should be based on the methods of the optimal control theory. Undoubtedly, it could be affirmed that optimization methods have not found proper application when considering the prospects for developing a single field, a group of fields or the prospects for developing the oil and gas industry.

In the present paper, an analysis of the goals and criteria on the problem of gas flows' optimal control in the framework of controlling the bottomhole pressure under certain conditions is performed.

The reasoning the choice of the technological mode of gas well operation is carried out. The formalization of the corresponding optimization problem is carried out, its reducibility to the problem of systems' optimal control is determined; the behavior of such systems can be described by a set of differential equations in partial and ordinary derivatives. An approach to its solution with the condition of the parameters' distribution is described.

The studies and analysis of the obtained numerical calculations allow us to draw the following conclusions:

1. Using the Pontryagin's maximum principle, which is a powerful mathematical apparatus, the study of optimal control problems allows us to determine the technological mode of gas well operation.

2. If the solution of the approximating system (12) – (13) with a fixed control $p = p_c(t)$ converges to the solution of the original boundary value problem, then there is always convergence in the functional, and the approximately optimal control found in this way is minimizing.

3. The gradient projection method with a specially selected step, even despite the incorrectness of the control problem for systems with quadratic functional, does not lead to a tendency to "hopper agitation", and in a short time gives a convergent minimizing sequence in the control space.

4. For the numerical solution of optimal control problems for nonlinear systems with distributed parameters, the use of the straight-line method is very effective, since it does not necessitate consideration of the conjugate boundary value problems for partial differential equations.

The results obtained determine the direction for further research, and the methods used may be useful for future research not only in the tasks of optimal control of gas wells, but also in other processes of designing the development of

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hydrocarbon deposits. The obtained data and conclusions can be included in studies on the development of optimal solutions for controlling gas wells.

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